Spontaneous emission of graviton by a quantum bouncer

G Pignol, K Protasov

Laboratoire de Physique Subatomique et de Cosmologie, CNRS/IN2P3-UJF-INPG, 53, Avenue des Martyrs, Grenoble, France

E-mail: `pignol@lpsc.in2p3.fr`

V Nesvizhevsky

Institute Laue Langevin, 6, rue Jules Horowitz, Grenoble, France

Abstract. Spontaneous emission of graviton rates for the quantum bouncer states are evaluated.

arXiv:quant-ph/0702256v1 27 Feb 2007

The quantum problem of a ball bouncing above a ideal mirror was considered a long time ago, however just as a mere exercise of elementary quantum mechanics. Things did change since the quantization of energy of Ultra Cold Neutrons (UCN) bouncing above a mirror in the Earth's gravitational field had been demonstrated in an experiment performed at the Institute Laue Langevin (ILL) [1, 2, 3]. However, this effect does not demonstrate any quantum behavior of the gravitational field itself. In analogy with electrodynamics, the observation of spectral lines in atoms shows the quantum behavior of electrons, but does not provide any clue concerning the possible quantization of the electromagnetic field. What does provide a clue is the observation of spontaneous decay of an excited state, for instance, which can only be explained in terms of photon emission.

So the observation of spontaneous decay of an excited state in the ILL experiment would be of interest, since it would be a Planck-scale physics effect. Nevertheless, the decay rate is expected to be low, and the purpose of this letter is to estimate it.

First we will set notations of the quantum bouncer problem, focusing on its physical implementation, that is, neutrons falling on the Earth's gravitational field above a perfect mirror. We then derive the spontaneous emission rate in a semi-classical approach.

The stationary Schrödinger equation for the vertical motion (z axis) of the mass m quantum bouncer is:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dz^2} + mgz\ \psi = E\ \psi. \tag{1}$$

The boundary condition due to the presence of the perfect mirror at $z = 0$ is $\psi(z = 0) = 0$. The characteristic length and the characteristic energy of this problem are:

$$z_0 = \left(\frac{\hbar^2}{2m^2 g}\right)^{1/3} = 5.87\ \mu\text{m} \tag{2}$$

$$E_0 = mgz_0 = 0.60\ \text{peV}. \tag{3}$$

The eigenproblem (1) can be solved in terms of the first Airy function $Ai(X)$, which has an infinite number of negative zeros, denoted by $\{-\lambda_1, -\lambda_2, \ldots\}$ in decreasing order. The energy of the stationary states is equal to $E_n = E_0\lambda_n$, and the wave function of the n^{th} state is:

$$\psi_n(z) = C_n Ai\left(\frac{z}{z_0} - \lambda_n\right)\ \theta(z) \tag{4}$$

where C_n normalizes the probability to find a neutron anywhere to 1. The sequence of the Airy function zeros has no simple analytic expression, but using the Bohr-Sommerfield rules, we find a fairly good approximation of this sequence:

$$\lambda_n \approx \left(\frac{3\pi}{8}(4n-1)\right)^{2/3}, \tag{5}$$

this approximation is known to be very good even for the lowest states.

In order to evaluate the rate for a bouncer to make a transition $k \to n$, we will follow the semi-classical analysis as in ref. [5]. This procedure consists in deriving the classical gravitational power P emitted by an oscillating quadrupole $Q\cos(\omega t)$, and by replacing the classical quadrupole by the quantum quadrupole moment for the transition $k \to n$ which reads

$$Q_{kn} = m\ \langle k|\hat{z}^2|n\rangle, \tag{6}$$

The quantum mechanical transition rate is:

$$\Gamma^{sp}_{k\to n} = \frac{P}{\hbar\omega_{kn}} = \frac{4}{15}\frac{\omega_{kn}^5}{M_{Pl}^2 c^4}\ Q_{kn}^2, \tag{7}$$

M_{Pl} is the Planck mass and $\omega_{kn} = (E_k - E_n)/\hbar$ is the angular frequency of the transition. This formula is valid if the quadrupole approximation $\omega_{kn}\ z_k \ll c$ holds, i.e. $k \ll 10^8$.

This semi-classical derivation for spontaneous emission of gravitons was recently considered in ref. [6] for atomic hydrogen. The authors provide a field theory derivation of Eq. (7) and show that it satisfies the detailed balance.

Since the quadrupole matrix elements for the quantum bouncer are known explicitly in terms of the Airy function zeros [7], we get:

$$\langle k|\hat{z}^2|n\rangle = \frac{24(-1)^{k-n+1}}{(\lambda_k - \lambda_n)^4} z_0^2. \tag{8}$$

Or, the transition probability is equal to

$$\Gamma^{sp}_{k\to n} = \frac{512}{5}\frac{1}{(\lambda_k - \lambda_n)^3}\left(\frac{m}{M_{Pl}}\right)^2 \frac{E_0^5\ z_0^4\ c}{(\hbar c)^5} = \frac{5\times 10^{-77}\text{s}^{-1}}{(\lambda_k - \lambda_n)^3}. \tag{9}$$

For the two lowest quantum states $\lambda_2 - \lambda_1 = 1,75$ and the probability of the spontaneous graviton emission is as low as:

$$\Gamma^{sp}_{2\to 1} \sim 10^{-77}\text{s}^{-1}. \tag{10}$$

References

[1] V. V. Nesvizhevsky *et al.*, Nature **415**, 297 (2002).
[2] V. V. Nesvizhevsky *et al.*, Phys. Rev. D **67**, 102002 (2003)
[3] V. V. Nesvizhevsky *et al.*, J. Phys. C **67**, (2005)
[4] T. Rothman and S. Boughn, Foundations of Physics **36**, (2006) 1801.
[5] S. Weinberg, "Gravitation and cosmology" (John Wiley, New York, 1972), chap. 10.
[6] S. Boughn and T. Rothman, Class. Quant. Grav. **23**, 5839 (2006)
[7] D. M. Goodmanson, Am. J. Phys. **68**, 866868 (2000).